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We present a short review of the recent 5D self-tuning solution of the cosmological constant problem with $1/H^2$ term, and present the dual description of the solution. In the dual description, we show that the presence of the coupling of the dual field(σ) to the brane(which is a bit different from the original theory) maintains the self-tuning property.

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I. INTRODUCTION

The cosmological constant problem is the most severe hierarchy problem or fine-tuning problem known to particle physicists since 1975 [1,2]. It was introduced in the Einstein equation as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - 8\pi G V_0 g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1}$$

where the term $8\pi G V_0$(V_0 is the vacuum energy density considered in particle physics.) is the so-called cosmological constant(c.c.) Λ. One can see that it is quite natural to introduce a constant in the action. Therefore, the constant is of order the mass scale in question. The parameter appearing in gravity is the Planck mass $M = 2.44 \times 10^{18}$ GeV which is astronomically larger than the electroweak scale. Since gravity introduces a large mass M, any other parameter in gravity is expected to be of that order, which is a natural expectation. Namely, V_0 is expected to be of order M. However, the bound on the vacuum energy was known to be $< (0.01\ \text{eV})^4$, which implied a fine-tuning of order 10^{-120}.

This c.c. problem surfaced as a very serious one when one considered the spontaneous symmetry breaking in particle physics [1]. The minimum position of potential does not matter in particle physics. But in gravity its position determines the c.c. There have been several attempts toward the solution, by Hawking [3], Witten [4], Weinberg [5], Coleman [6], etc, under the name of probabilistic interpretation in Euclidian gravity, boundary of different phases, anthropic solution, wormhole solution, etc.

The self-tuning solution, which attracted a great deal of attention recently, is basically that the equations of motion choose the vanishing cosmological constant solution. For example, if the flat space solution exists, then it is a zero cosmological constant solution. In 4D, a nonzero vacuum energy does not allow the flat space solution. Thus, to obtain the flat space one has to fine-tune V_0 to zero very accurately.

In Sec. II, we review the self-tuning ideas. In Sec. III, we summarize the self-tuning solution with H_{MNPQ} discussed in Ref. [7,8]. In Sec. IV, we present a self-tuning solution in the dual description in terms of σ. This self-tuning property is shown to be maintained even though we introduce the brane coupling of the dual field σ. Sec. V is a conclusion.

II. SELF-TUNING IDEAS

We can distinguish the self-tuning solutions into the old version and the new version.

A. Old version

If there exists a solution for the flat space, then the existence itself is called a self-tuning solution, or a solution with the undetermined integration constant(UIC). For a nonzero Λ in 4D, a flat space ansatz $ds^2 = d\mathbf{x}^2 - dt^2$ does not allow a solution. The de Sitter space(dS, $\Lambda > 0$) or anti de Sitter space(AdS, $\Lambda < 0$) solution is possible. To reach a nearly flat space solution, one needs an extreme fine tuning, which is the c.c. problem.

But suppose that there exists an UIC. Witten used the four index field strength $H_{\mu\nu\rho\sigma}$ to obtain an UIC [4]. The equation of motion of H leads to an UIC, say c. Thus, the vacuum energy contains a piece $\sim c^2$. This UIC c can be adjusted so that the final c.c. is zero. Once c is determined, there is no more UIC because $H_{\mu\nu\rho\sigma}$ is not a dynamical field in 4D. When vacuum energy is added later, there is no handle to adjust further. In a sense, it was another way of fine-tuning. However, if there exists a dynamical field allowing an UIC, it is a desired old style self-tuning solution. This old version did not care whether there also exist de Sitter or anti de Sitter solutions. Selection of the flat space out of these solutions is from a principle such as Hawking's probabilistic choice.

B. New version

In recent years, a more ambitious attempt was proposed, where only the flat space ansatz has the solution [9].(But at present there does not exists a solution in this category.) The Randall-Sundrum II type models [10] were constructed in 5D bulk AdS, i.e. the 5D bulk cosmological constant $\Lambda_b < 0$, with brane(s) located at the fixed point at $y = 0$. The RS II model uses only one brane. At this brane one can introduce a brane tension Λ_1. Thus, the gravity Lagrangian contains two free parameters Λ_b and Λ_1. In Fig. 1, we show this situation schematically, where the extra dimension is the y-direction. x^μ is the 4D index. The 4D flat space ansatz allows a solution for a specific choice of k_1(basically Λ_1, $k_1 = \Lambda_1/6M^3$) and k(basically Λ_b, $k = \sqrt{-\Lambda_b/6M^3}$): $k_1 = k$. Therefore, it requires a fine-tuning between parameters as in the 4D case. However, it seems to be an improvement since we reach at the 4D flat space from nonzero cosmological constants and the RS II type models seems to be a good playground to obtain UIC solutions.

The RS II model is an interesting extension of the space-time without compactification. With the bulk AdS, the uncompactified fifth dimension can be acceptable due to the localized gravity [10]. For the brane located at $y = 0$, the action is

$$S = \int d^4x \int dy \sqrt{-g}\bigg(\frac{M^3}{2}(R - \Lambda_b) + (-\Lambda_1 + \mathcal{L}_{matter})\delta(y)\bigg) \tag{2}$$

where M is the 5D fundamental scale and $\mathcal{L}_{matter}$ is the matter Lagrangian, assuming the matter is located at the brane only. The flat space ansatz,

$$ds^2 = \beta(y)^2 \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \tag{3}$$

allows the solution if $k = k_1$. Even though the 4D is flat, it is curved in the direction of the fifth dimension, denoted by the warp factor $\beta(y) = \beta_0 \exp(-k|y|)$. Namely, the gravity is exponentially unimportant if it is separated from the brane. [If there were more branes, there are more conditions to satisfy toward a flat space since one can introduce a brane tension at each brane. Thus, the RS II type models are the simplest ones.]

The try to obtain a new type of self-tuning solution was initiated by Arkani-Hamed et. al. and Kachru et. al. [9]. For example the 5D Lagrangian with the introduction of a massless bulk scalar field ϕ, coupling to the brane tension,

$$\mathcal{L} = R - \Lambda e^{a\phi} - \frac{4}{3}(\nabla\phi)^2 - Ve^{b\phi}\delta(y) \tag{4}$$

where we set $M^3/2 = 1$. We may ask, "Why this Lagrangian?", which involves more difficult related questions. Accepting this, we must satisfy the following Einstein and field equations, with the flat ansatz (3),

$$\begin{gathered}
(\text{dilaton}) : \frac{8}{3}\phi'' + \frac{32}{3}A'\phi' - a\Lambda e^{a\phi} - bV\delta(y)e^{b\phi} = 0 \\
(55) : 6(A')^2 - \frac{2}{3}(\phi')^2 + \frac{1}{2}\Lambda e^{a\phi} = 0 \\
(55), (\mu\nu) : 3A'' + \frac{4}{3}(\phi')^2 + \frac{1}{2}e^{b\phi}V\delta(y) = 0
\end{gathered} \tag{5}$$

where $2A(y) = \ln \beta(y)$, and prime denotes the derivative with respect to y.

We will discuss the property of this solution somewhat in detail, to compare with our new solution obtained in the dual picture.

For $\Lambda = 0$, there exists a bulk solution satisfying $A' = \alpha\phi'$,

$$\phi = \pm\frac{3}{4}\ln\left|\left(\frac{4}{3}\right)y + c\right| + d, \quad \alpha = \pm\frac{1}{3} \tag{6}$$

where c and d are determined without fine-tuning of the parameters. The solution has a singularity at $y_c \equiv -(3/4)c$ or diverges logarithmically at large $|y|$. The logarithmically diverging solution does not realize the localization of gravity. If we restrict the space up to the singular point y_c, then at every y inside the space it is flat. However, the effective 4D theory is the one after integrating out the allowed y space. Since y_c is the naked singularity, we do not know how to cut the y integration near y_c, implying a possibility that the flat space ansatz does not lead to a solution. Depending on how to cut the integral, one may introduce a nonzero c.c. Förste et. al. tried to understand this problem by curing the singularity by putting a brane at y_c [11]. Then, a flat 4D space solution is possible but one needs a fine-tuning. It is easy to understand. If one more brane is introduced, then there is one more tension parameter Λ_2, i.e. in the Lagrangian one adds $\Lambda_2\delta(y - y_c)$. If the space is flat for one specific value of Λ_2, then it must be curved for the other values of Λ_2, since the y integration gives a c.c. contribution directly from Λ_2.

This example teaches us that the self-tuning solution better should not have a singularity in the whole y space. Even if we neglect the problem of singularity, the brane interaction Eq.(4) must have *a specific value for b, which is a fine-tuning.* Thus, we have not obtained a new self-tuning solution yet.

III. THE SELF-TUNING SOLUTION WITH $1/H^2$

As we have seen in the RS II model, the Einstein-Hilbert action alone does not produce a self-tuning solution. Inclusion of higher order gravity does not improve this situation [12]. We need matter field(s) in the bulk. The first try is a massless spin-0 field in the bulk as Ref. [9] tried so that it affects the whole region of the bulk. However, it may be better if there appears a symmetry in the spin-0 sector. These are achieved by a three index antisymmetric tensor field A_{MNP}. In 5D the dual of its field strength is interpreted as a scalar. The field strength H_{MNPQ} is invariant under the gauge transformation $A_{MNP} \to A_{MNP} + \partial_{[M}\lambda_{NP]}$, thus masslessness arises from the symmetry. There will be one $U(1)$ gauge symmetry remaining with one massless pseudoscalar field which is a, $\partial_M a = (1/4!)\sqrt{-g}\epsilon_{MNPQR}H^{NPQR}$. But the interactions are important for the solution, as in Ref. [9] a bulk solution was found for the specific form of the interaction.

The first guess is the bulk term $-(M/48)H^2$ [8] where $H^2 \equiv H_{MNPQ}H^{MNPQ}$. The brane with tension Λ_1 is located at $y = 0$, and the bulk c.c. is Λ_b. The ansatze for the solution are

$$\begin{aligned}\text{Ansatz 1} &: ds^2 = \beta(y)^2\eta_{\mu\nu}dx^\mu dx^\nu + dy^2,\\ \text{Ansatz 2} &: H_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma}\frac{\sqrt{-g}}{n(y)}\end{aligned} \tag{7}$$

where $\mu, \cdots$ are the 4D indices, and $n(y)$ is a function of y to be determined. It is sufficient to consider (55) and $(\mu\nu)$ components Einstein equations and the H field equation. By setting $M = 1$, we obtain the bulk solution [8]

$$\begin{aligned}\Lambda_b < 0 &: \beta(|y|) = (\frac{a}{k})^{1/4}[\pm\sinh(4k|y| + c)]^{1/4}\\ \Lambda_b > 0 &: \beta(|y|) = (\frac{a}{k})^{1/4}[\sin(4k|y| + c')]^{1/4}\\ \Lambda_b = 0 &: \beta(|y|) = |4a|y| + c''|^{1/4}.\end{aligned} \tag{8}$$

For a localizable (near $y = 0$) metric, there exists a singularity at $y = -c/4k$, etc., except for some cases with $\Lambda_b > 0$. Thus, for these singular cases another brane is necessary to cure the singularity, and we need a fine-tuning as in the case of Kachru et. al. [9]. The solution with the bulk de Sitter space without a singularity, the second one in Eq. (8), is worth commenting. Such a solution is periodic and depicted in Fig. 2. We can consider only $|y| \le y_c$, then $\beta' = 0$ at $y = \pm y_c$. The boundary condition at $y = 0$ determines $c' = \cot^{-1}(k_1/k)$ and the boundary condition at y_c determines y_c such that $c' = 4ky_c - \cot^{-1}(k_2/k)$, so it looks like an UIC. Indeed, Ref. [13] claims that such a solution is a self-tuning one. But for y_c to behave like an undetermined integration constant, it should not appear in the equations of motion. Note, however, that y_c is the VEV of the radion g_{55}, and hence it cannot be a strictly massless Goldstone boson. If it were massless, it will serve to the long range gravitational interaction and hence give different results from the general relativity predictions in the light bending experiments. Therefore, it should obtain a mass and hence y_c is not a free parameter but fixed. So the boundary condition at y_c is a fine-tuning condition [8,14].

A working self-tuning model is obtained with the $1/H^2$ term [7],

$$S = \int d^4x \int dy\sqrt{-g}\left(\frac{1}{2}R + \frac{2\cdot 4!}{H^2} - \Lambda_b - \Lambda_1\delta(y)\right). \tag{9}$$

A. Flat space solution

For the flat space ansatze, we use

$$\begin{aligned}ds^2 &= \beta(y)^2\eta_{\mu\nu}dx^\mu dx^\nu + dy^2,\\ H_{\mu\nu\rho\sigma} &= \epsilon_{\mu\nu\rho\sigma}\frac{\sqrt{-g}}{n(y)}, \quad H_{5\mu\nu\rho} = 0.\end{aligned} \tag{10}$$

The H field equation is $\partial_M[\sqrt{-g}H^{MNPQ}/H^4] = \partial_\mu[\sqrt{-g}H^{\mu NPQ}/H^4] = 0$, and hence fixes n as a function of y only. The two relevant Einstein equations are

$$\begin{aligned}(55) &: 6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b - \frac{\beta^8}{A}\\ (\mu\nu) &: 3\left(\frac{\beta'}{\beta}\right)^2 + 3\left(\frac{\beta''}{\beta}\right) = -\Lambda_b - \Lambda_1\delta(y) - 3\left(\frac{\beta^8}{A}\right)\end{aligned} \tag{11}$$

where $\Lambda_b < 0$ and $2n^2 = \beta^8/A$ with $A > 0$. We require the Z_2 symmetry, and the bulk equation is easily solved. The boundary condition at $y = 0$ is $(\beta'/\beta)|_{0^+} = -\Lambda_1/6$. Then, we find a solution

$$\beta(|y|) = \frac{1}{\left[(\frac{a}{k})\cosh(4k|y| + c)\right]^{1/4}} \tag{12}$$

where

$$k = \sqrt{\frac{-\Lambda_b}{6}}, \quad a = \sqrt{\frac{1}{6A}}, \quad k_1 = \frac{\Lambda_1}{6}. \tag{13}$$

The flat solution is shown in Fig. 3.

This solution has the integration constants a and c. a is basically the charge of the universe and determines the 4D Planck mass. c is the UIC which is fixed by the boundary condition at $y = 0$,

$$\tanh(c) = \frac{k_1}{k} = \frac{\Lambda_1}{\sqrt{-6\Lambda_b}}. \quad (14)$$

This solution shows that, for any value of Λ_1 in the finite region allowing tanh, it is possible to have a flat space solution. Even if the observable sector adds some constant to Λ_1, still it is possible to have the flat space solution, just by changing the shape a little bit via c. The change is acceptable since H is a dynamical field. Note that $\beta(y)$ is a decreasing function of $|y|$ in the large $|y|$ region and it goes to zero exponentially as $|y| \to \infty$. We propose that this property is needed for a self-tuning solution.

The key points found in our solution are

(i) β has no singularity: Our solution extends to infinity without singularity, and $\beta' \to 0$ as $y \to \infty$.

(ii) 4D Planck mass is finite: Even if the extra dimension is not compact, this theory can describe an effective 4D theory since gravity is localized. Integrating with respect to y, we obtain an effective 4D Planck mass which is finite

$$\begin{aligned} M^2_{4D\ Planck} &= \int_{-\infty}^{\infty} dy \beta^2 \\ &= 2M^3 \frac{k}{a} \int_0^{\infty} \frac{1}{[\cosh(4ky+c)]^{1/2}} dy \end{aligned} \quad (15)$$

and is expected to be of order the fundamental parameters.

(iii) Self-tuning: We obtained a self-tuning solution. To check that the 4D c.c. is zero we integrate out the solution. For that we have to include the surface term also

$$S_{surface} = \int d^4x dy 4\cdot 4! \partial_M \left[\sqrt{-g} \frac{H^{MNPQ} A_{NPQ}}{H^4} \right]. \quad (16)$$

Then the action is

$$\begin{aligned} S = \int d^4x dy \sqrt{-g_4} \beta^4 \Bigg[& \frac{1}{2\beta^2} R_4 - 4\frac{\beta''}{\beta} - 6\left(\frac{\beta'}{\beta}\right)^2 \\ & - \Lambda_b + \frac{2\cdot 4!}{H^2} - \Lambda_1 \delta(y) \Bigg] + S_{surface}. \end{aligned} \quad (17)$$

Then, the effective 4D c.c. term $-\Lambda_{4D}$ is the y integral except the R_4 term. One can show that $\Lambda_{4D} = 0$ and it is consistent with the original ansatz of the flat space [7].

B. De Sitter and anti de Sitter space solutions

For the de Sitter and anti de Sitter space solutions, the metric is assumed as

$$\begin{aligned} & ds^2 = \beta(y)^2 g_{\mu\nu} dx^\mu dx^\nu + dy^2 \\ & g_{\mu\nu} = diag.\left(-1, e^{2\sqrt{\lambda}t}, e^{2\sqrt{\lambda}t}, e^{2\sqrt{\lambda}t}\right), (dS_4, \lambda > 0) \\ & g_{\mu\nu} = diag.\left(-e^{2\sqrt{-\lambda}x_3}, e^{2\sqrt{-\lambda}x_3}, e^{2\sqrt{-\lambda}x_3}, 1\right), (AdS_4, \lambda < 0). \end{aligned} \quad (18)$$

Note that $k = \sqrt{-\Lambda_b/6}, k_1 = \Lambda_1/6$, and the 4D Riemann tensor is $R_{\mu\nu} = 3\lambda g_{\mu\nu}$. The (55) and (00) components equations are

$$\begin{aligned} & 6\left(\frac{\beta'}{\beta}\right)^2 - 6\lambda\frac{1}{\beta^2} = -\Lambda_b - 3\frac{\beta^8}{A} \\ & 3\left(\frac{\beta'}{\beta}\right)^2 + 3\frac{\beta''}{\beta} - 3\lambda\frac{1}{\beta^2} = -\Lambda_b - \Lambda_1\delta(y) - 3\frac{\beta^8}{A}. \end{aligned} \quad (19)$$

The 4D c.c. obtained from the above ansatze is λ. Since we cannot obtain the solution in closed forms, we cannot show this by integration. However, we have checked this kind of behavior [8] in the RS II model, using the Karch-Randall form [15]. Here, we show just that the de Sitter and anti de Sitter space solutions exist, and show the warp factor numerically. In our model, the y derivative of the metric is

$$\beta' = \pm(k_\lambda^2 + k^2\beta^2 - a^2\beta^{10})^{1/2}, \quad k_\lambda = (\lambda/6)^{1/2} \quad (20)$$

At $y = y_h$ where $\beta(y_h) = 0$, $\beta'(y_h)$ needs not be zero due to the presence of the nonvanishing k_λ. Therefore, there exists a point y_h where β' is finite. It is the de Sitter space horizon. It takes an infinite amount of time to reach y_h. Also, we can see that it is possible that β can be nonzero where β' is zero for $k_\lambda^2 < 0$. It is the anti de Sitter space solution. These de Sitter space and anti de Sitter space solutions are shown in Figs. 4 and 5. For the de Sitter space, we can integrate from $-y_h$ to $+y_h$. As in the Karch-Randall example, it should give the 4D c.c. λ. The AdS solution does not give a localized gravity.

In the presence of de Sitter and anti de Sitter space solutions, the c.c. problem relies on the old self-tuning solution. Namely, the c.c. is probably zero, following Hawking [3].

Hawking showed that in 4D the probability is maximum, using the Euclidian space action

$$-S_E = -\frac{1}{2}\int d^4x\sqrt{+g}[(R+2\Lambda) + (1/48)H_{\mu\nu\rho\sigma}H^{\mu\nu\rho\sigma}] \quad (21)$$

where we use the unit $8\pi G = 1$. The Einstein equation and field equations are

$$\begin{aligned} & R_{\mu\nu} - (1/2)g_{\mu\nu}R = \Lambda g_{\mu\nu} - T_{\mu\nu} \\ & \partial_\mu[\sqrt{g}H^{\mu\nu\rho\sigma}] = 0. \end{aligned}$$

From the field equation, one has $H^{\mu\nu\rho\sigma} = (1/\sqrt{g})\epsilon^{\mu\nu\rho\sigma}c$ or $H^2 = 4!c^2$. Thus, he obtained

$$T^{\mu\nu} = -\Lambda_H g^{\mu\nu}, \Lambda_H = -c^2/2,$$
$$R = -4\Lambda_{eff}, \Lambda_{eff} = \Lambda + \Lambda_H. \tag{22}$$

Thus,

$$-S_E = -(1/2)\int d^4x\sqrt{g}(R + 2\Lambda_{eff})$$
$$= \text{Volume} \cdot (\Lambda + \Lambda_H) = \frac{3M^4}{\Lambda_{eff}} \tag{23}$$

which is maximum at $\Lambda_{eff} = 0^+$ which is shown in Fig. 6.

Note that Hawking used equations of motion. Duff, on the other hand, used the action itself to calculate the Euclidian action, and obtained $-S_E = 3M^4(\Lambda - (3/2)c^2)/(\Lambda - (1/2)c^2)^2$ which is minimum at 0^+ [16]. But the consideration of the surface term in the action would give additional contribution and should give Hawking's result. The surface term is essential as we have shown in our self-tuning solution.

Thus, the maximum probability occurs when the c.c. is zero. Our self-tuning solution relies on this probabilistic choice of the flat one from the flat, de Sitter and anti de Sitter space solutions.

IV. SELF-TUNING SOLUTION IN THE DUAL DESCRIPTION

Let us now proceed to consider a dual description of the self-tuning solution with $1/H^2$. When we introduce the $1/H^2$ term in the Lagrangian, the equation of motion for a three form field A with H=dA is*

$$\mathrm{d}\left(\frac{{}^*\mathrm{H}}{(H^2)^2}\right) = 0 \tag{24}$$

where *H is the Hodge dual of H. Note that we have not introduced a source term for A_{MNP}.

The corresponding field strength H=dA should satisfy the Bianchi identity, dH=0, locally. In the dual description, the roles of the equation of motion and the Bianchi identity are interchanged from the original one. Therefore, we can take a dual of the field strength H in 5D such that the equation of motion for A is trivially satisfied, as a Bianchi identity in the dual description, as follows:

$$\mathrm{d}\sigma = \frac{{}^*\mathrm{H}}{(H^2)^2} \tag{25}$$

where σ is a dual scalar field. Then, we can identify the field strength H in terms of the scalar field σ as

$$H_{MNPQ} = \sqrt{-g}\epsilon_{MNPQR}\frac{\partial^R\sigma}{(4!)^{1/3}[(\partial\sigma)^2]^{2/3}}. \tag{26}$$

As a result, the original Bianchi identity, dH=0, becomes in the dual picture

$$\partial_M\left(\sqrt{-g}\frac{\partial^M\sigma}{[(\partial\sigma)^2]^{2/3}}\right) = 0,. \tag{27}$$

which will be the equation of motion in the dual picture. In this case, we can add a source term on the right hand side of Eq. (27).

To show the self-tuning solution with $1/H^2$ in the dual picture explicitly, let us insert the dual relation (25) or (26) back to the Lagrangian and make a variation of the action with respect to the dual scalar field . When the surface term for A is included, the dual 5D action is

$$S = \int d^4xdy\,\sqrt{-g}\left(\frac{1}{2}R - \gamma[(\partial\sigma)^2]^{1/3} - \Lambda_b - \Lambda_1 f(\sigma)\delta(y)\right) \tag{28}$$

where $\gamma = 6(4!)^{4/3}$ and $f(\sigma)$ is an arbitrary coupling of the dual field to the brane, which breaks the shift symmetry of the dual field at the brane.

Then, the energy-momentum tensor from the dual scalar field is

$$T^\sigma_{MN} = 2\gamma\left(\frac{1}{3}[(\partial\sigma)^2]^{-2/3}\partial_M\sigma\partial_N\sigma - \frac{1}{2}g_{MN}[(\partial\sigma)^2]^{1/3}\right). \tag{29}$$

And the equation of motion for the dual scalar field becomes

$$\partial_M\left(\sqrt{-g}\frac{\partial^M\sigma}{[(\partial\sigma)^2]^{2/3}}\right) = \sqrt{-g^{(4)}}\Lambda_1\frac{df}{d\sigma}\delta(y) \tag{30}$$

where the original Bianchi identity (27) is not respected at the brane due to the coupling $f(\sigma)$, i.e.,

$$dH_{5\mu\nu\rho\sigma} = \frac{1}{(4!)^{1/3}}\sqrt{-g^{(4)}}\epsilon_{\mu\nu\rho\sigma}\Lambda_1\frac{df}{d\sigma}\delta(y). \tag{31}$$

Note that the self-tuning solution with $1/H^2$ is the case with $\frac{df}{d\sigma}$=0. In the dual picture, a new Bianchi identity

*The text mode(Roman) characters are the form notation while equation mode(Italic) characters are the component notation.

is dd$\sigma = 0$. Thus, the theory we study is different from the original one in that the bulk field σ couples to the brane with $f(\sigma)$.

If we take the ansatze for the metric and the scalar field as

$$ds^2 = \beta^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad \sigma = \sigma(y), \tag{32}$$

then the relevant Einstein's equations and the scalar equation are

$$3\left(\frac{\beta'}{\beta}\right)^2 + 3\frac{\beta''}{\beta} = -\Lambda_b - \Lambda_1 f(\sigma)\delta(y) - \gamma|\sigma'|^{2/3}, \tag{33}$$

$$6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b - \frac{1}{3}\gamma|\sigma'|^{2/3}, \tag{34}$$

$$\left(\beta^4\frac{\sigma'}{|\sigma'|^{4/3}}\right)' = \beta^4\Lambda_1\frac{df}{d\sigma}\delta(y). \tag{35}$$

Thus, we find that the solutions of the above three equations consistent with the Z_2 symmetry become

$$\beta(y) = \left(\frac{k}{a}\right)^{1/4}[\cosh(4k|y| + c)]^{-1/4}, \tag{36}$$

$$\sigma'(y) = \pm\left(\frac{18a^2}{\gamma}\right)^{3/2}\beta^{12}\epsilon(y) \tag{37}$$

where $\epsilon(y)$ is the step function which is $+1$ for $y > 0$ and -1 for $y < 0$, and

$$k = \sqrt{-\frac{\Lambda_b}{6}}, \quad a = \sqrt{\frac{1}{6A}}. \tag{38}$$

On the other hand, from Eqs. (33) and (35), the boundary conditions at the brane for the metric and the scalar are given by

$$\left.\frac{\beta'}{\beta}\right|_{y=0^+} = -k_1 f(\sigma(0)), \tag{39}$$

$$\left.\frac{\sigma'}{|\sigma'|^{4/3}}\right|_{y=0^+} = 3k_1\frac{df}{d\sigma}(\sigma(0)) \tag{40}$$

where

$$k_1 = \frac{\Lambda_1}{6}. \tag{41}$$

Therefore, there arise two consistency conditions due to the existence of the brane and the scalar coupling :

$$\tanh(c) = \frac{k_1}{k}f(\sigma(0)), \tag{42}$$

$$\pm\sqrt{\frac{\gamma}{2}}\frac{1}{k}\cosh(c) = 9k_1\frac{df}{d\sigma}(\sigma(0)). \tag{43}$$

That is, the condition for the scalar coupling function at the brane becomes

$$\frac{df}{d\sigma}(\sigma(0)) = \pm\sqrt{\frac{\gamma}{162}}\frac{1}{kk_1}\left(1 - \left(\frac{k_1}{k}\right)^2 f^2(\sigma(0))\right)^{-1/2}. \tag{44}$$

Without the scalar coupling, Eq. (43) is absent and Eq. (42) is the self-tuning solution obtained in terms of the three form field [7], and we reproduce the solution in the dual picture.

Even with the scalar coupling, i.e. the original Bianchi identity becoming the equation of motion in the dual picture with the scalar coupling to the brane, the self-tuning property is maintained because $\sigma(0)$ acts as an integration constant of Eq. (37). For a given set of Λ_b, Λ_1, and parameters appearing in $f(\sigma)$, Eq. (44) can be satisfied by choosing an appropriate value of the integration constant $\sigma(0)$. For example, if $f(\sigma) = e^{b\sigma}$ then for some b we can find $\sigma(0)$ as far as Eq. (44) is satisfied. Note that the condition Eq. (44) is given only for the field value at $y = 0$. Of course, there also exist de Sitter and anti de Sitter space solutions. Thus, our self-tuning solution is of the old style.

To see it explicitly, let us consider $f(\sigma)$ as an exponential function

$$f(\sigma) = e^{b\sigma} \tag{45}$$

where we set the overall constant as 1 and b is a number. Then, Eq. (44) becomes

$$b^2 e^{2b\sigma(0)}\left[1 - \frac{k_1^2}{k^2}e^{2b\sigma(0)}\right] = \frac{\gamma}{162}\frac{1}{(kk_1)^2} \tag{46}$$

which is a quadratic equation of $e^{2b\sigma(0)}$. For a finite range of parameters, it has two solutions, proving the self-tuning property. It is different from the Kachru et al. solution which required a fine-tuning of b as $4/3$.

V. CONCLUSION

It is pointed out that a self-tuning solution of the cosmological constant is possible in the RS II type models. We have shown this $1/H^2$ term in the Lagrangian as a review to guide the self-tuning solution [7,8]. We presented this solution in the dual picture. The kinetic energy term in the dual picture has a peculiar form $((\partial_\mu\sigma)^2)^{1/3}$. In the dual picture, this self-tuning property is still maintained even with the presence of the dual field coupling to the brane. However, the new type of self-tuning solution is not obtained. The self-tuning relies on the old idea of choosing the flat one out of numerous possibilities [3].

The dual picture description is intriguing. As discussed in Sec. IV, the scalar field σ has an integration constant $\sigma(0)$. We can choose $\sigma(0)$ such that a flat space solution results. However, for another nearby value of $\sigma(0)$, a de Sitter or anti de Sitter space solution is possible, which is the reason that we could have not obtained a new type of self-tuning solution. The unique flat space solution requires a fine-tuning of $\sigma(0)$, at present. In the future, it will be interesting to see if some principle chose such a fine-tuning of $\sigma(0)$.

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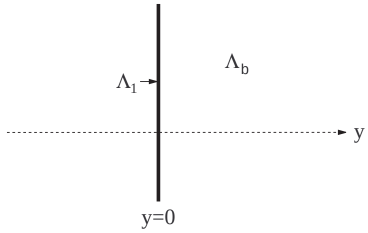

Fig. 1. The RS II model with a brane at $y = 0$.

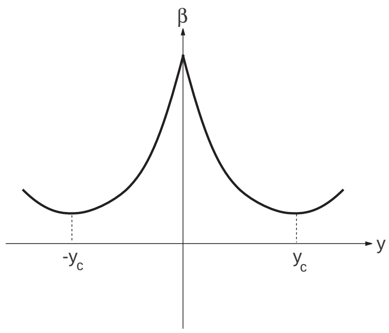

Fig. 2. The flat space solution with H^2

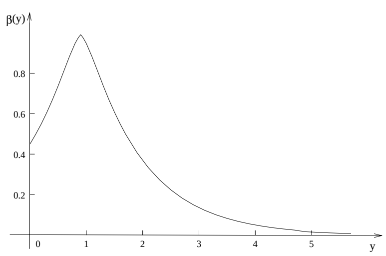

Fig. 3. The flat space solution.

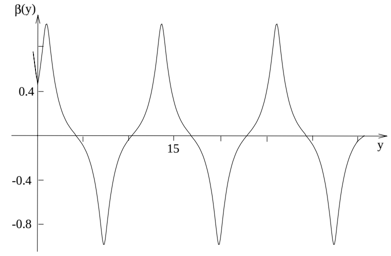

Fig. 4. The de Sitter space solution.

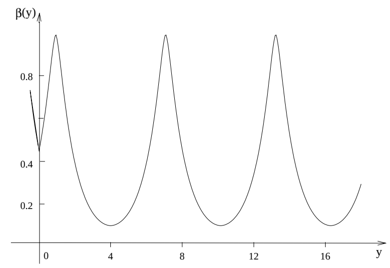

Fig. 5. The anti de Sitter space solution.

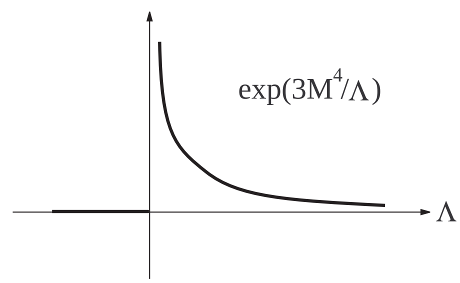

Fig. 6. Hawking's probability